

Question: - (a) State and Explain the (basic) fundamental postulates of the special theory of relativity.

(b) A reference frame  $S$  moves w.r. to another frame  $S'$  with uniform velocity  $\vec{v}$ . Derive Lorentz space and Time Transformation eq<sup>n</sup> gives  $x', y', z', t'$  in terms of  $x, y, z, t$  the moving frame coincides with stationary ones at  $t'=t=0$ . Prove that when  $\vec{v}$  is much smaller than velocity of light Lorentz Transformation reduce to Galilean Transformations.

Ans: - (a) Special theory of relativity :->

The special theory of relativity was enunciated in 1905 by Albert-Einstein. It has two fundamental postulates: -

1) The laws of physics are invariant in all inertial systems :- An inertial system is defined as a co-ordinate frame of reference within which the law of inertia, i.e. Newton's first law of motion holds. A body  $m$  which no net external force acts will move with a uniform velocity, if it is in an inertial system. Hence according to this postulates the mathematical form of physical law remains the same for any two observers moving with constant linear velocity relative to each other is therefore, not possible to distinguish one inertial system from another by an experiment in physics, As the laws of physics are the same for an inertial frame. In other words there is no preferred inertial system.

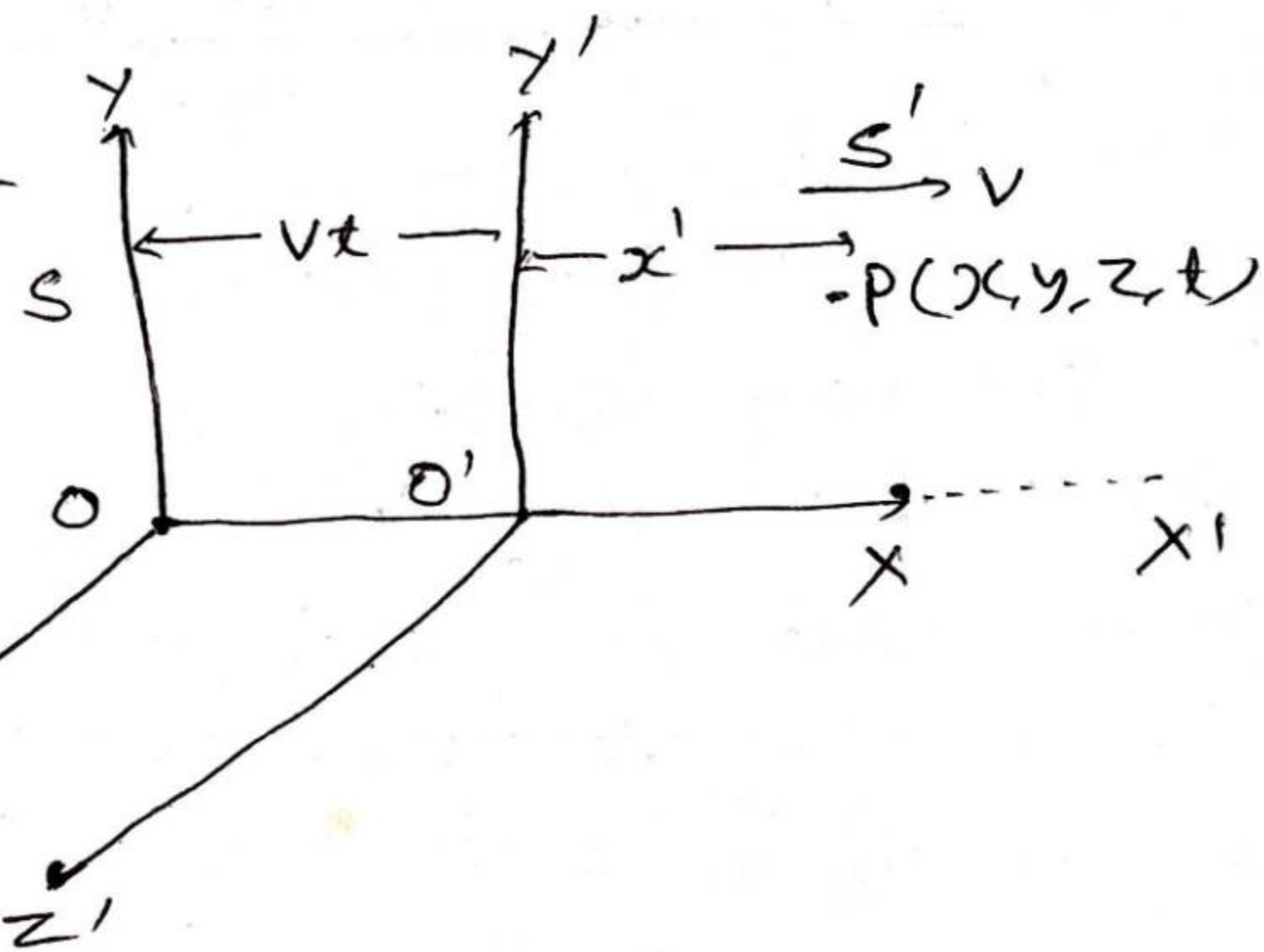
The speed of light in vacuum is a constant independent of the inertial system, the source and the observer.

In other words, the velocity of light is an invariant.

Systems moving with a uniform velocity relative to one another are called Lorentz transformation.

Consider two observers  $O$  and  $O'$  located in two separate inertial coordinate systems  $S$  and  $S'$ . The system  $S'$  moves with a uniform velocity  $v$  to the right along the  $x$ -axis relative to  $S$ . This is equivalent to the motion of  $S$  to the left with a velocity  $v$  relative to  $S'$ . Suppose each observer carries a metre rod and a clock to measure the position and time of a particle relative to an inertial system. By specifying the position and time of a physical phenomenon, the observer describes, what is called an event.

The space and time co-ordinates of an event at  $P$  described by an observer are  $(x, y, z, t)$  and the co-ordinates of the  $x, y, z$  give distance from the origin  $O$  along the  $x, y$  &  $z$  directions measured by the meter stick of the observer  $O$ .  $t$  gives the time



he reads on his clock. Suppose both the observers are temporarily at rest w.r. to each other when they compare their metre sticks and synchronise their clocks. The systems then set in motion w.r. to System  $S$ . When the origin  $O'$  passes the origin of  $S$  both the clocks reads zero, and  $t' = 0$  and at that instant  $x = x'$ .

It is evident that after time  $t$  as measured by  $O$ , the origin of the system  $S'$  is at distance  $vt$  from the origin of the space  $S$ .

$$\therefore x' = x - vt$$

As the relative velocity  $v$  is small compared to the speed of light  $c$ , the Lorentz transformation reduces to Galilean transformation.

Similarly for observer  $O'$  in system  $S'$ ,  $c = \sqrt{x'^2 + y'^2 + z'^2} / t'$  (4)

$$\therefore x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0$$

from (1) and (11)  $x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c^2 t'^2$ ,  $y = y'$  &  $z = z'$  (11)

$$\therefore x^2 - c^2 t^2 = x'^2 - c^2 t'^2$$

Substituting  $x = Ax' + Bt'$  and  $t = Gx' + Ht'$  we get

$$(Ax' + Bt')^2 - c^2 (Gx' + Ht')^2 = x'^2 - c^2 t'^2$$

Comparing co-efficients on both sides we have

$$x'^2 [A^2 - c^2 G^2] = x'^2 \quad \text{or} \quad A^2 - c^2 G^2 = 1 \quad \text{--- (III)}$$

$$t'^2 [B^2 - c^2 H^2] = -c^2 t'^2 \quad \text{or} \quad B^2 - c^2 H^2 = -c^2 \quad \text{--- (IV)}$$

$$2x't' [AB - c^2 GH] = 0 \quad \text{or} \quad AB - c^2 GH = 0 \quad \text{--- (V)}$$

Now when  $x' = 0$ ,  $x = vt$

$$\therefore x = Ax' + Bt' \quad \text{and} \quad t = Gx' + Ht', \quad \text{we get} \quad vt = Bt' \quad \& \quad t = Ht'$$

$$\therefore vHt' = Bt' \quad \therefore B = vH$$

from (IV)  $v^2 H^2 - c^2 H^2 = -c^2$  or  $H^2 = \frac{c^2}{c^2 - v^2}$   $\therefore H = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Substituting this value of  $H$  in  $B = vH$ , we get  $B = \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}}$

from (V)  $B = vH$  (we get)

$$AvH - c^2 GH = 0 \quad \text{or} \quad Av = c^2 G \quad \therefore G = \frac{Av}{c^2}$$

Substituting  $G = Av/c^2$  in eq<sup>n</sup> (III)

$$A^2 - A^2 \frac{v^2}{c^2} = 1 \quad \therefore A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{Hence} \quad G = \frac{v/c^2}{\sqrt{1 - v^2/c^2}}$$

Substituting these values of  $A, B, G$  &  $H$ , the Lorentz transformation are

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = t' + \frac{v}{c^2} \frac{x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y = y' \quad \text{and}$$